Mitigation of vibrations by stiff wave barriers: physical mechanisms

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Introduction

Stiff wave barrier next to the track

- Construction of a jet grouting wall next to the track
- Wave impeding barrier for railway induced vibrations

- Common construction techniques:
  - deep vibro compaction
  - gravel/cement columns
  - hydraulic fracture injection with stable cement–bentonite mixtures
  - ...
**Introduction**

**Stiff wave barrier next to the track**

- Block of stiffened soil in a homogeneous halfspace (e.g. by means of jet grouting)

<table>
<thead>
<tr>
<th></th>
<th>$C_s$ [m/s]</th>
<th>$C_p$ [m/s]</th>
<th>$\beta_s$ [-]</th>
<th>$\beta_p$ [-]</th>
<th>$\rho$ [kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Halfspace</td>
<td>200</td>
<td>400</td>
<td>0.025</td>
<td>0.025</td>
<td>2000</td>
</tr>
<tr>
<td>Stiffened soil</td>
<td>550</td>
<td>950</td>
<td>0.05</td>
<td>0.05</td>
<td>2000</td>
</tr>
</tbody>
</table>

- Square cross-section: $w = h = 2$ m
Transfer functions

- Real part of the vertical displacement field $\hat{u}_z(x, \omega)$ at 5 Hz
  
  without barrier
  
  with barrier

- Corresponding insertion loss $\hat{IL}_z(x, \omega) = 20 \log_{10} \left( \frac{|\hat{u}_z^{\text{ref}}(x, \omega)|}{|\hat{u}_z(x, \omega)|} \right)$
Transfer functions

- Real part of the vertical displacement field $\hat{u}_z(x, \omega)$ at 30 Hz
  without barrier
  with barrier

- Corresponding insertion loss $\hat{IL}_z(x, \omega) = 20 \log_{10} \frac{|\hat{u}_z^{\text{ref}}(x, \omega)|}{|\hat{u}_z(x, \omega)|}$
Transfer functions

- Real part of the vertical displacement field $\hat{u}_z(x, \omega)$ at 60 Hz

  without barrier

  with barrier

- Corresponding insertion loss $\hat{IL}_z(x, \omega) = 20 \log_{10} \frac{|\hat{u}_{z}^{\text{ref}}(x, \omega)|}{|\hat{u}_z(x, \omega)|}$
Transfer functions

Plane wave propagation

- Cylindrical wavefield can be decomposed into plane waves, satisfying the dispersion relation
  \[ \frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} = \frac{1}{\lambda_R^2}, \]
  where \( \lambda_R = 2\pi \frac{C_R}{\omega} \) is the Rayleigh wavelength.

- Propagating plane waves are characterized by \( \lambda_R \leq \lambda_y \leq \infty \):
  - \( \theta = 0 \Rightarrow \lambda_x = \lambda_R, \lambda_y = \infty \)
  - \( \theta = \pi/2 \Rightarrow \lambda_y = \lambda_R, \lambda_x = \infty \)
Transfer functions

Plane wave propagation

- Rayleigh wave propagation: \( \frac{1}{\lambda_x} + \frac{1}{\lambda_y} = \frac{1}{\lambda_{Rt}} \)
- (a) Without and (b) with stiff wave barrier:

\(- \lambda_y = \infty (\lambda_x = \lambda_R) (\theta = 0)\)

\(- \lambda_R \leq \lambda_y \leq \infty (\theta = 0.50)\)
Transfer functions

Plane wave propagation

• Rayleigh wave propagation: \( \frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} = \frac{1}{\lambda_{RT}^2} \)

• (a) Without and (b) with stiff wave barrier:

  - \( \lambda_R \leq \lambda_y \leq \infty \) (\( \theta = 1.2 \))

  - \( \lambda_y < \lambda_R \) (\( \theta = \pi/2 \))
Transfer functions

Interaction of Rayleigh waves in the soil and bending waves in the stiff wave barrier

• Rayleigh wave dispersion curve (black line):
  \[ \lambda_R = 2\pi \frac{C_R}{\omega} \]

• Euler-Bernoulli beam theory in \((\lambda_y, \omega)\)-domain:
  \[ \left( -\rho A \omega^2 + EI \left( \frac{2\pi}{\lambda_y} \right)^4 \right) \tilde{u}_z(\lambda_y, \omega) = \tilde{f}(\lambda_y, \omega) \]

• Free bending wave dispersion curve (red line):
  \[ \lambda_b = \frac{2\pi}{\sqrt{\omega}} \left( \frac{EI}{\rho A} \right)^{1/4} \]

\[ \Rightarrow \tilde{u}_z(\lambda_y, \omega) \propto 0 \text{ for } \lambda_y < \lambda_b \]

\[ \lambda_y = \infty \]

\[ \lambda_y = 2/3 \lambda_b \]

\[ \lambda_y = \lambda_b \]

\[ \lambda_y = 1/3 \lambda_b \]

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Transfer functions

Interaction of Rayleigh waves in the soil and bending waves in the stiff wave barrier

- \( f < f_c: \lambda_b < \lambda_R \Rightarrow \) Rayleigh wave propagates unhindered through the block of stiffened soil
- \( f > f_c: \lambda_b > \lambda_R \Rightarrow \) wavefield is partially transmitted, partially blocked
  - \( \lambda_y > \lambda_b: \) plane waves are transmitted
    \( \tilde{IL}_z(x, \omega) \sim 0 \text{ dB} \)
  - \( \lambda_y < \lambda_b: \) transmission of plane waves is impeded by the block of stiffened soil

- **Critical frequency** \( f_c \) (intersection of the Rayleigh wave and the free bending wave dispersion curves):

  \[
  f_c = \frac{\omega_c}{2\pi} = \frac{C_R^2}{2\pi} \sqrt{\frac{\rho A}{EI}} = \frac{C_R^2}{2\pi h} \sqrt{\frac{12\rho}{E}} = 12 \text{ Hz}
  \]
Transfer functions

Interaction of Rayleigh waves in the soil and bending waves in the stiff wave barrier

• The propagating plane waves $\lambda_y > \lambda_R$ are characterized by a wave propagation direction $\theta = \sin^{-1}(\lambda_R/\lambda_y)$. A reduction of vibration levels in the spatial domain will only be obtained in an area delimited by a critical angle $\theta_c(\omega) = \sin^{-1}(\lambda_R/\lambda_b)$:

$$\sin \theta_c = \frac{C_R}{\sqrt{\omega}} \left( \frac{\rho A}{EI} \right)^{1/4} = \frac{C_R}{\sqrt{\omega h}} \left( \frac{12\rho}{E} \right)^{1/4}$$

\[\text{Diagram showing the interaction of waves and critical angle}\]